1.

ON A THEOREM IN THE GEOMETRY OF POSITION.

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WE propose to apply the following (new?) theorem to the solution of two problems in Analytical Geometry.

Let the symbols

$$|\alpha|, \quad \begin{vmatrix} \alpha, & \beta \\ \alpha', & \beta' \end{vmatrix}, \quad \begin{vmatrix} \alpha, & \beta, & \gamma \\ \alpha', & \beta', & \gamma' \\ \alpha'', & \beta'', & \gamma'' \end{vmatrix}, \text{ \&c.}$$

denote the quantities

$$\alpha,\ \alpha\beta' - \alpha'\beta,\ \alpha\beta'\gamma'' - \alpha\beta''\gamma' + \alpha'\beta''\gamma - \alpha'\beta\gamma'' + \alpha''\beta\gamma' - \alpha''\beta'\gamma, \text{ \&c.}$$

(the law of whose formation is tolerably well known, but may be thus expressed,

$$|\alpha| = \alpha, \quad \begin{vmatrix} \alpha, & \beta \\ \alpha', & \beta' \end{vmatrix} = \alpha\,|\beta'| - \alpha'\,|\beta|,$$

$$\begin{vmatrix} \alpha, & \beta, & \gamma \\ \alpha', & \beta', & \gamma' \\ \alpha'', & \beta'', & \gamma'' \end{vmatrix} = \alpha \begin{vmatrix} \beta', & \gamma' \\ \beta'', & \gamma'' \end{vmatrix} + \alpha' \begin{vmatrix} \beta'', & \gamma'' \\ \beta, & \gamma \end{vmatrix} + \alpha'' \begin{vmatrix} \beta, & \gamma \\ \beta', & \gamma' \end{vmatrix}, \text{ \&c.}$$

the signs + being used when the number of terms in the side of the square is odd, and + and − alternately when it is even.)

Then the theorem in question is

$$\begin{vmatrix} \rho\,\alpha + \sigma\,\beta + \tau\,\gamma.., & \rho\,\alpha' + \sigma\,\beta' + \tau\,\gamma'.., & \rho\,\alpha'' + \sigma\,\beta'' + \tau\,\gamma''.. \\ \rho'\alpha + \sigma'\beta + \tau'\gamma.., & \rho'\alpha' + \sigma'\beta' + \tau'\gamma'.., & \rho'\alpha'' + \sigma'\beta'' + \tau'\gamma''.. \\ \rho''\alpha + \sigma''\beta + \tau''\gamma.., & \rho''\alpha' + \sigma''\beta' + \tau''\gamma'.., & \rho''\alpha'' + \sigma''\beta'' + \tau''\gamma''.. \\ \vdots & \vdots & \vdots \end{vmatrix} = \begin{vmatrix} \rho, & \sigma, & \tau.. \\ \rho', & \sigma', & \tau'.. \\ \rho'', & \sigma'', & \tau''.. \\ \vdots & \vdots & \vdots \end{vmatrix} \begin{vmatrix} \alpha'', & \beta'', & \gamma''.. \\ \alpha', & \beta', & \gamma'.. \\ \alpha, & \beta, & \gamma.. \\ \vdots & \vdots & \vdots \end{vmatrix}$$